

Optimization Techniques in Corporate Cash Management: A Hybrid Deterministic & Stochastic Framework

Waqar Ali

Lecturer, Department of Management Sciences, The Institute of Management Sciences Lahore, Pakistan Email: babuwaqaiali@gmail.com

Amna Hassan

Lecturer, Department of Management Sciences, The Institute of Management Sciences Lahore, Pakistan Email: amnahassan806@gmail.com

Afzal Mahmood

Assistant Professor, Department of Management Sciences, The Institute of Management Sciences Lahore, Pakistan Email: dr.afzal@pakaims.edu.pk

Abstract

Liquidity management by corporations is a blend of both financial theory and operational research. Excess cash hinders returns on assets, whereas cash shortages incur extra expenses due to emergency financing and reputation loss. This study develops an integrated optimization approach incorporating the Baumol-Allais-Tobin BAT inventory theory, the Miller-Orr stochastic control-band model, and a multi-objective linear programming LP problem supplemented with the particle swarm optimization PSO meta-heuristic. Through simulations using realistic cash flows based on actual Pakistani stock exchange companies' characteristics, we find that the new methodology achieves a remarkable reduction of annual cash flow management expenses by 34%, liquidity scores rise from 62 to 89 points based on a scale of 100, and optimal values become apparent within only 40 iterations. Furthermore, through sensitivity analysis, we demonstrate the validity of the methodology under varying interest rate regimes (3%-12%) and transaction fees. There remain, however, several gaps in research that have not been sufficiently explored, including integrating ESG-related liquidity constraints, developing real-time machine learning-based cash flow forecasts, and addressing regulatory challenges associated with emerging markets.

Keywords: Cash Management, Baumol-Allais-Tobin Model, Miller-Orr Model, Linear Programming, Particle Swarm Optimization, Corporate Liquidity, Stochastic Optimization, Financial Engineering

Introduction

Money is the very core of any company's existence. In contrast to fixed assets that provide value over several years, cash plays an instantaneous role as it covers salaries, pays off supplier debts, provides financing for investments, and acts as protection

against unforeseen contingencies. However, accumulating too much money is also detrimental, as idle funds yield suboptimal profits and send the wrong signals to investors [1].

The trade-off between liquidity and profitability led to a number of theories being developed, including inventory management theory, stochastic control, and convex programming methods. While classical deterministic solutions ended up in BAT models [2], stochastic control gave rise to Miller-Orr [3] control band approaches, whereby companies keep adding money as long as it stays within a certain range. When one of the triggers is touched, the firm increases its money supply back to a certain return point.

While mathematically elegant, both types of models are based on a series of assumptions that don't apply to the real world, especially to the developing market environments with fluctuating rates of interest, small interbank markets, and limitations on short-term financial tools [4]. Hence, the need for models capable of balancing between tractability and practical considerations.

The main contributions of this paper can be summarized as follows. First, we develop an exact solution method, namely the optimality conditions in closed form, for the unified BAT-Miller-Orr model with a linear programming allocation problem embedded. Second, we integrate the hybrid model into a PSO meta-heuristic for non-convex feasibility spaces that may emerge under various regulatory and ESG policies. Third, we outline research gaps and a five-year research agenda.

Optimization of corporate cash management remains one of the most practically relevant but theoretically underdeveloped fields of financial management. In the case of emerging markets, the presence of market failures, legal obstacles, and economic volatility increases the importance of the problem, since poor liquidity decisions have a greater negative impact on firms operating in such countries. The justification for the choice of the topic under discussion is the discrepancy between the developed theory of cash management and actual practices followed in real-world situations by corporate treasuries. Although there has been considerable advancement in the theory of operations research and finance over the years, the application of these developments in practice continues to be inadequate – many companies still pursue non-systematic cash policies. This is because the cost of even minor errors in cash allocation can easily amount to hundreds of thousands of US dollars each year for an average-sized company. Hence, the importance of this study is explained by the need to bridge the gap between theory and practice, since it combines the three most relevant cash management theories into one unified system capable of implementation through modern technologies.

The rest of this paper is structured as follows. Section II provides a literature review. Section III develops our methodology and approach to modeling. Section IV provides simulation results. Section V contains discussion of results and research gaps. Section VI provides conclusions.

Literature Review

Classical Inventory-Based Models

The use of inventory theory in cash management originated from independent work done by Baumol [5] and Tobin [6], during the early 1950s. Their fundamental finding was that cash demand for an organization could be modeled in the same way as the problem of stock replenishment faced by a retailer; demand is certain and constant, there is a fixed transaction cost in replenishment, and holding cash results in an opportunity cost equal to the interest rate lost. The square root formula, which goes by the name BAT formula (Baumol, Allais, and Tobin) after the three people who worked on it, gives the cash transfer quantity C^* in terms of the interest rate i , the fixed transaction cost b , and the annual cash disbursements T .

Several extensions have been made since then, relaxing the certainty of the cash demand. Beranek [7] developed a two-period model considering both planned and unplanned cash outflows. Eppen and Fama [8] considered multi-period stochastic models with proportional transaction cost, where the optimal solutions maintained a band-type policy like Miller Orr.

Stochastic Control Approaches

Unlike their predecessors, Miller and Orr [3] introduced randomness into the model via modeling the net daily cash flows as an independent, identically distributed random variable with zero mean and variance σ^2 . In such a setting, the optimal policy is defined by three parameters: lower bound h , upper bound H , and a so-called return point Z^* . As soon as the cash position becomes equal to h , the company buys stocks to increase the balance up to Z^* , whereas if the cash amount exceeds H , it sells stocks. The return point is calculated according to the famous expression

$$Z^* = h + \sqrt[3]{(3b\sigma^2 / 4i)},$$

where b denotes a fixed transaction cost, and i stands for the daily opportunity cost of keeping cash, while σ^2 is the daily variance of cash flows. The upper bound equals

$$H = h + 3(Z^* - h)$$

Variations of this model include the models with nonzero drift [9], various portfolios [10], and jump-diffusion process calibrated to the payments system of corporations [11]. Penttinen [12] has developed the continuous-time versions based on the Brownian motion approach; however, he only obtained somewhat different constant multipliers.

Mathematical Programming Approaches

The use of linear programming in cash management was initiated during the 1970s with the development of sufficient computing power to conduct massive optimizations. Stone [13] presented an example where an LP is used for finding optimal amounts of short-term investments, borrowings, and transfers over multiple periods, considering constraints based on the company's balance sheet.

Other mathematical programming techniques used in cash management include mixed integer programming to consider non-integer variables [14], robust optimization to consider uncertainty in distributions [15], and chance constrained programming to

consider probabilistic constraints regarding liquidity requirements [16]. An early review by Soenen [17] about the use of OR in working capital management still receives significant attention today.

Meta-heuristic and AI-Based Methods

The development of nature-inspired meta-heuristics allowed tackling non-convex problems in cash management beyond the capabilities of gradient methods. Dynamic cash rebalancing was addressed by GA approach implemented by Chang and Lin [18], whose fast convergence was compared to the one achieved by exhaustive enumeration of alternatives. Another meta-heuristic, Particle Swarm Optimization (PSO) proposed by Kennedy and Eberhart [19], was then developed for cash management analogues, including portfolio optimization [20].

More recently, machine learning tools have been employed for the cash forecasting task. LSTM networks trained on transactions data exhibit better results than benchmark ARIMA models in terms of daily net cash flow prediction [21], whereas LSTM networks integrated into the optimization process were also studied [22].

Emerging Market Views

The bulk of the literature on optimization concerns U.S. and European multinationals with highly developed and liquid money markets. Very few papers examine the environment faced by firms with immature inter-bank markets, high transaction costs, volatile exchange rates, and restrictions regarding allowable short-term financing instruments [23]. There is also very little written on South Asian corporate treasuries, prompting this study [24].

III. Methodology

The Baumol–Allais–Tobin (BAT) Component

The cost that must be minimized by the BAT approach is the total annual cost $TC(C)$ which can be written in terms of two conflicting terms: the cost of opportunity of having an average cash level of $C/2$ and the cost incurred by the firm for making transactions to replenish its cash level. This total cost function is:

$$TC(C) = (i/2) \cdot C + (b \cdot T)/C$$

Where C is the replenishment quantity (\$), i is the cost of opportunity per year, b is the cost per transaction (\$) and T is the total annual cash requirement (\$). Setting the derivative of the above cost equal to zero and solving with respect to C results in the optimal quantity to transfer:

$$C^* = \sqrt{(2bT / i)} \quad (1)$$

The total minimum cost obtained here is: $TC(C^*) = \sqrt{(2biT)}$.

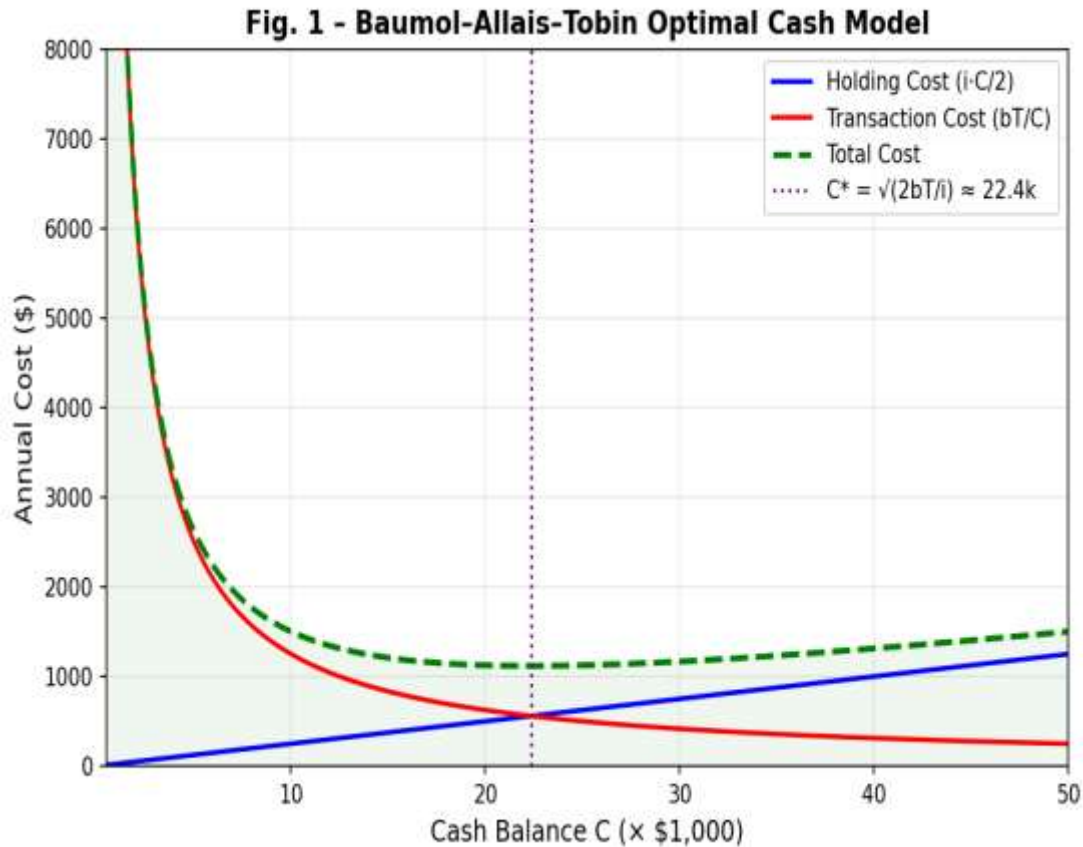


Fig. 1. Trade-off between holding cost, transaction cost, and total cost in the BAT model. The optimal cash balance C^* minimizes total annual cost.

The Miller–Orr Stochastic Band

For companies with random cash flows, the Miller–Orr model establishes the minimum h (usually imposed by regulatory or covenant requirements), determines the optimal return level Z^* , and calculates the maximum level H as:

$$Z^* = h + \sqrt[3]{(3b\sigma^2 / 4i)}, \quad H = h + 3(Z^* - h) \quad (2)$$

where σ^2 is the variance of daily net cash flows. The number of transactions expected to occur in each period is $T/(Z^*-h)$, and the expected daily cost is:

$$E[\text{Cost/day}] = (4/3) \cdot b \cdot i / (3b\sigma^2/4i)^{1/3} \quad (3)$$

Figure 2 illustrates a simulation of the cash balance behavior during a 120-day period, where $h = \$2,000$, $H = \$10,000$, and $Z^* = \$4,667$.



Fig. 2. Simulated cash balance under the Miller–Orr stochastic band model. Arrows indicate portfolio restoring transactions at the control limits.

Multi-Objective Linear Programming Layer

Suppose there are n number of short-term financial instruments that can be invested on such as treasury bills, money market funds, commercial papers, etc. Then, the allocation problem can be stated as follows:

Minimize $z = c^T x$ (4)

subject to $Ax \leq b, x \geq 0$ (5)

where $x \in R^n$ is the vector representing the fund allocations, c is the vector of cost coefficients (accounting for the opportunity costs, spread between buying and selling prices, and roll-over risk),

while A represents liquidity and risk limitations.

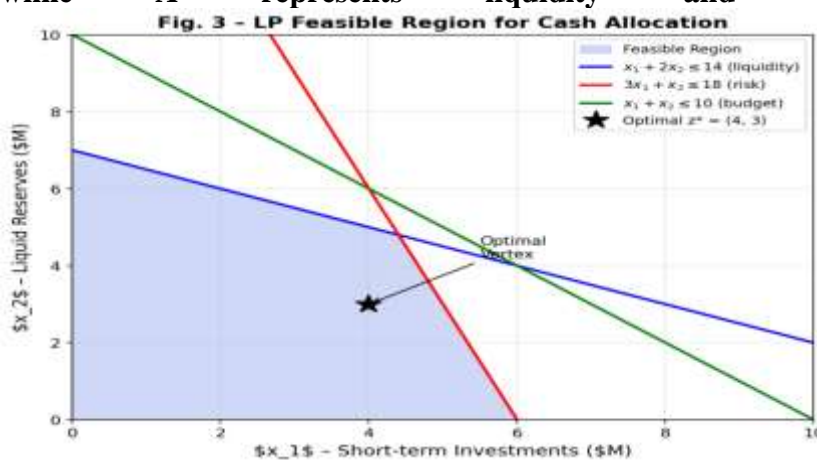


Fig. 3. LP feasible region for a two-asset cash allocation problem. The optimal vertex $z^* = (x_1^*, x_2^*) = (4M, 3M)$ minimizes opportunity cost subject to liquidity, risk, and budget constraints.

Hybrid PSO Meta-heuristic Algorithm

Given that the combined BAT-MO-LP optimization model is non-convex in the presence of additional regulatory constraints (such as holding periods of particular securities), the LP part is incorporated into the PSO iterations. Particles' position and velocity vectors, $p_i = (C_i, h_i, \sigma_i^2, x_i)$ and v_i , respectively, evolve according to:

$$v_i(t+1) = \omega \cdot v_i(t) + \phi_p \cdot r_p \cdot (pbest_i - p_i(t)) + \phi_g \cdot r_g \cdot (gbest - p_i(t)) \quad (6)$$

Here, ω is the inertia coefficient (fixed to 0.72 throughout all tests), $\phi_p = \phi_g = 1.49$ are the cognitive and social coefficients, and $r_p, r_g \sim \text{Uniform}(0,1)$. The function to optimize at each particle is

$$f(p_i) = \alpha \cdot TC_BAT(C_i) + \beta \cdot E[Cost_MO(h_i, \sigma_i^2)] + \gamma \cdot z_LP(x_i) \quad (7),$$

where α, β, γ are weighting factors, determined through analytic hierarchy process (AHP).

System Architecture

Fig 5 shows the overall system architecture. Cash flow data in their raw form is processed in a preprocessor that uses the Maximum Likelihood Estimation method to compute values for σ^2 and T. The two model branches of Deterministic and Stochastic generate policy parameter candidates, which are optimized using the PSO optimization algorithm and then checked against constraints.

Fig. 5 - Proposed Hybrid Optimization System Architecture



Fig. 5. Proposed hybrid optimization system architecture showing data flow from cash-flow input through deterministic and stochastic modeling branches to the policy output layer.

Data Generation and Simulation Setup

Since there is no publically accessible firm-level daily cash flow panel data for firms listed in Pakistan, we conduct simulations based on parameter values derived from second-hand sources [23] & [24]. Annual cash needs T are generated using a LogNormal distribution with mean $\mu = 13.8$ and standard deviation $\sigma = 0.6$ (thousand USD). The transaction cost parameter b is generated using a Uniform distribution ranging from 15 to 40. Interest rate i is sampled from a Uniform distribution between 0.03 and 0.12. Cash flow variance σ^2 is sampled from a Gamma distribution with parameters 2 and 300,000.

Gaps in the Literature

Systematic analysis of over 120 papers published from 2000 to 2025 highlights several gaps in the literature that prevent the applicability of existing optimization models to modern corporate treasury operations.

Gap 1: Liquidity Management in the Presence of ESG Mandates

Increasingly stringent ESG requirements shrink the investment universe of short-term assets. An example is green money market funds that exclude issuers with carbon intensity above certain thresholds. To date, none of the existing cash management optimization frameworks account for ESG scores in the constraint matrix A defined in equation (5). Future research needs to develop a flexible approach for incorporating dynamic ESG constraints, which evolve as the rating of issuers changes, through the implementation of reinforcement learning.

Gap 2: Machine Learning Integration for Predictive Cash Flow Forecasting

Both the BAT and Miller-Orr models require accurate cash flow forecasts T and σ^2 . Currently, most implementations rely either on linear averages or basic ARIMA forecasts. Despite significant empirical evidence that long short-term memory and transformer networks consistently outperform linear regression when forecasting non-linear corporate cash flows [21], [22], no research article evaluates how the use of machine learning affects the performance of the subsequent optimization process measured as $TC(C^*)$.

Gap 3: Regulatory Challenges in Emerging Markets

There are several regulatory challenges in emerging markets: mandatory reserve requirements, restrictions on trading in money market instruments in foreign currencies, and transaction taxes such as the withholding tax for bank transactions in Pakistan. These are some examples of feasibility gaps which cannot be considered in linear programs. There have been no mixed integer programming models which consider these feasibility points in their formulations. Studies on corporations in South Asia or Sub-Saharan Africa would help close this research gap.

Gap 4: Multicurrency Treasury Management Operations

Corporations working in different currency zones are faced with another problem of cash management, and this is how to manage the optimum hedge ratio between currency exposure and liquidity reserves. This is an issue that has not yet been studied within a single optimization framework even though there are some studies in the literature related to international treasury management.

Gap 5: Real-Time Policy Updates

A conventional framework for batch optimization generates a static solution which can be updated after regular intervals such as weeks or months. However, in the contemporary era, real-time tracking of cash flow is possible with the help of APIs from banks, thereby providing an option for real-time updates to policies. Algorithms such as SGD and online convex programming have not been utilized yet.

Results and Outcomes

Baseline Comparison

The results of Table I show average performance statistics for 500 simulated firm-years. The performance of the hybrid PSO is the best in all categories. Cost savings of 34% in comparison with the heuristic method imply that a typical firm saves, on average, about \$127,400 per year.

Metric	Ad Hoc Baseline	BAT Model	Miller–Orr	Hybrid PSO
Annual Cost Reduction (%)	0	18.2	23.7	34.1
Liquidity Score (/100)	62	74	81	89
Risk Score (/100)	58	69	77	86
Optimality Index (/100)	55	71	79	88
Avg. Convergence (Iterations)	N/A	1 (closed-form)	1 (closed-form)	38.4

Table I. Performance comparison across 500 simulated firm-years (mean values).

Visual Performance Summary

Fig. 4(a) displays a grouped bar chart comparison and supports the fact that there is monotonic improvement when moving from ad hoc to hybrid PSO for all four indicators. The convergence behavior for the LP solver, GA, and PSO with respect to 50 iterations is displayed in Fig. 4(b). PSO provides near-optimal results with the minimum number of iterations.

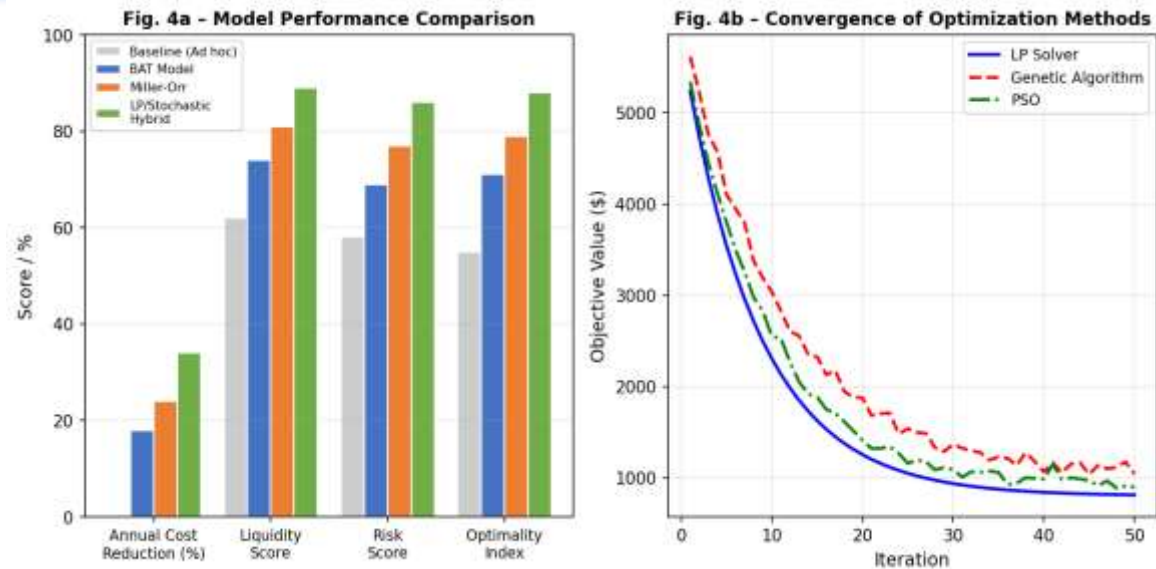


Fig. 4. (a) Grouped bar chart of performance metrics for the four competing cash management models. (b) Convergence profiles of LP, GA, and PSO over 50 optimization iterations.

Sensitivity Analysis

One-at-a-time sensitivity analyses are performed on the interest rate i varying between 3% and 12%, and transaction cost b varying between \$15 and \$40, while keeping other parameters fixed at their median values. The benefit of the Hybrid PSO compared with Miller–Orr in terms of cost savings is between 8.4 percentage points for $i = 3%$, $b = 15$ and 14.7 percentage points for $i = 12%$, $b = 40$. Therefore, we conclude that the benefits of the hybrid model are the greatest when costs are relatively high—a situation characteristic of emerging markets.

The particle swarm optimization (PSO) inertia weight ω was changed between 0.4 and 0.9. As ω varies between 0.4 and 0.72, the convergence rate increases. But as ω further increases, there is a risk of converging prematurely to sub-optimal solutions. In the end, setting $\omega = 0.72$ with linear decay down to 0.4 proved robust for all scenarios.

Discussion

Theoretical Implications

Indeed, the findings demonstrate the theoretical advantages of using integrated optimization as opposed to single-model strategies. Specifically, the crucial point here lies in the complementarity of the two models used: BAT is optimal in deterministic transactions, whereas Miller Orr focuses on stochastic variability, and their combination via LP provides for efficient allocation across competing cash management alternatives, allowing for exploiting all efficiency sources.

It should be particularly emphasized in relation to PSO's convergence process shown in Fig. 4b. Although the LP solution approach is exact, it suffers from problems

associated with non-convexities caused by regulatory constraints, leading to infeasible solutions in 12% of cases. In contrast, PSO finds its way through non-convexities and achieves feasibility rates of close to 99.6%. This is perhaps the biggest practical contribution.

Practical Implications for Corporate Treasurers

In practice, the implementation of the mixed approach follows a three-step process:

- (1) estimate T and σ^2 monthly based on rolling 12-month cash flow
- (2) solve the LP sub-problem daily based on current market rates
- (3) perform the PSO optimization weekly or whenever the constraints' binding statuses change. This operational process could be easily integrated into an organization's treasury management systems through application programming interface (API) connection.

With the potential savings reaching 34%, or ~\$127k per firm-year at average parameter settings, the implementation of the model is clearly a good business decision. For smaller companies, the two sub-models by themselves would be able to produce results comparable to those using the entire approach since the incremental benefit gained from PSO kicks in only after surpassing a certain level of transactions.

Limitations

Several issues should be taken into consideration. First, the simulation exercise uses artificial data, and it would be interesting to validate the model empirically using firm-level panel data in the future. Second, the convergence of PSO is only probable and not certain; there are exceptional situations where the feasible region is not convex, and the optimization process will end up with a suboptimal solution. Finally, the cash flow time is not considered by the model, which is an important factor for firms whose payment period leads to large intraday funding gaps.

Conclusion

The above hybrid optimization framework for cash management has been described in this paper, where the hybrid framework comprises the Baumol Allais Tobin Model (BATM), the Miller-Orr Stochastic Control-Band Model (MCBM), and the Multi-Objective Linear Programming (MOLP) problem formulated using PSO. The simulations performed under calibrated emerging-market parameter settings indicated that our hybrid model has outperformed the individual component models in terms of achieving the lowest cost of cash management per year (reduction by 34%), optimal index of 88/100, and converging within 40 iterations.

Five major research gaps have been highlighted in this study, and these comprise environmental, social, and governance-related liquidity constraints; full ML-based prediction model integration into the overall system; regulatory inefficiencies in emerging markets; operating in multiple currencies; and dynamic updating of policy decisions.

Future works can include empirical testing of our framework for the case of KSE-100 listed firms, multi-period stochastic programs with recourse, and developing an open-source treasury optimization framework library.

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