

Green Bonds Connectedness with International Financial Markets: Dynamics of Time-Frequency Optimization towards Green Portfolio

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Abstract

Green bonds, which cumulative have issued over \$370 billion in 2023, represent a relatively new and increasingly popular type of investment not only in terms of financial performance, but also in its environmental impact and, accordingly, have provided you with a large portion of your sustainable investment portfolio. The study determines how green bonds and traditional asset classes in 10 years between 2014 and 2024 have changing dynamic interdependencies and reveals the time-varying correlation patterns of the two groups under both stable and distressed market conditions. The study provides the research with the support of a long historical time frame, which in turn supports the singular diversification merits and risk-adjusted resilience of green bonds and offers the key information on the timing of the allocation as well as the portfolio construction rationale to the investors. This study can contribute to the comprehension of the role of green bonds in the process of transition to a sustainable financial system, and it can be shown that these bonds can play a stabilizing role in financial markets at the time of financializing and instabilizing the situation in the world. The study insights will be crucial for policy-making, institutional investment and market participants to enhance asset allocation and promote sound low-carbon finance systems. Through more efficient regulatory systems and the development of sustainable finance programs, green bonds can play major role in taking the world towards a greener, more environmentally aware global economy.

Keywords: Financial markets, Wavelet coherence, Sustainable investment, Green Bond, Economic stability, Economic crises, Sustainable financial systems.

1. INTRODUCTION

The emerging nature of green investing is manifested in green bond-traditional finance interdependence, where the investment approach and the sustainable finance intersect (Khan & Richman, 2026). The climate risk is accumulating on an international scale, and the green bonds securities, which are used to finance the green projects, have become very popular with the issuance level reaching more than 370 billion as of 2023 (Kumar et al., 2025). Their use in portfolios aimed at financial and environmental performance can be identified as growth drivers as researchers and investors are trying to determine the interdependence between the green bonds its classic asset holdings, i.e., equities and corporate bonds (Hasan et al., 2024; *Interdependence between Green Financial Instruments and Major Conventional Assets: A Wavelet-Based Network Analysis*, n.d.-a). The significance of the topic lies not just in the fact that this allows to diversify the portfolio and minimize risk but specifies the profile of stability of green bonds in the case of being under the financial stress situation.

The empirical facts have shown that green financial products tend to correlate highly with traditional assets in distressed markets, pointing towards the use of green bonds as a stabilizer in the case of financial stress (*Interdependence between Green Financial Instruments and Major Conventional Assets: A Wavelet-Based Network Analysis*, n.d.-b) (*Incorporating Green Bonds into Portfolio Investments: Recent Trends and Further Research*, n.d.) (*ESG Perceptions: Investigating Investor Motivations and Characteristics* | *Financial Services Review*, n.d.) These results hold special importance because investors are looking for methods of maximizing asset allocation while keeping with sustainable investment objectives and transformation (Aloui et al., 2026; Do & Nguyen, 2025). But the integration of green bonds into “traditional” portfolios is controversial. However, critics state that there is not enough evidence to suggest the effectiveness of the green bonds as safe haven asset, and the potential for 'greenwashing' (misleading investments as green investments) is an issue to consider with regards to investor confidence and regulatory control.

(*Extreme Connectedness between Green Bonds, Government Bonds, Corporate Bonds and Other Asset Classes: Insights for Portfolio Investors*, n.d.; *In Charts: Green and Sustainability Bonds*, n.d.) With the market of green bonds growing, it is important to learn how it relies on larger financial markets so that strategies to develop the robust investment based on sustainability but not return-raising can be developed. Conclusively, the study of the dependence structure of green bonds and international financial markets can be used to inform portfolio management, improve risk assessment and provide policy implications to support a sustainable financial ecosystem. With the increasing demand of green investments, more studies will be required on such relationships to make sure that green bonds contribute to major roles in meeting world environmental goals (Castellanos, 2024) (*Green Bonds in Portfolio Diversification*, 2024) (*Frontiers* | *Dynamic Relationship between Green Bonds and Major Financial Asset Markets from the Perspective of Climate Change*, n.d.).

One of the most significant themes of research is the interdependence between the green financial instruments such as the green bonds traditional asset in the context of the growing significance of sustainable investments. In this section, interdependence is

tested using complicated techniques, e.g., wavelet coherence and network analysis, to establish how interdependence varies over time horizons and frequencies (*Eco-Friendly Investing | Benefits, Key Principles & Challenges*, n.d.).

Wavelet analysis, a method that came into view in the mid 1980s, is most applicable when it comes to the analysis of interactions between time series at different frequencies and time scales (Batra et al., 2026). The approach uses zero mean and normalized wavelets in order to perform an analysis of financial time series. The most widespread is the Morlet wavelet as it is useful to detect local correlations in both time and frequency (Martiradonna et al., 2023). One of the measures of the local linear relationship between pairs of signals is squared wavelet coherence, which gives a more delicate view of co-movement between them than less subtle protocols in the time domain. This solution eliminates the flaws of the Pearson correlation coefficient that only measures the direction of relationship and not the strength of relationship at different frequencies. This answer avoids the weaknesses of the Pearson correlation coefficient in capturing only direction of association but not varying strength of relation at different frequencies. (*The Eligibility of Green Bonds as Safe Haven Assets: A Systematic Review*, n.d.).

To link environmentally sustainable financial products to conventional asset classes through networks, distance measures based on correlation coefficient are employed. The study distinguishes the overall sampling period into sub-periods that are most significant to study, for instance, during European sovereign debt crisis, oil price declines, and COVID-19 crisis. Through such division, one can attain a more precise idea of the change of interdependence due to economic shocks. The findings show that the relationship between the green financial instruments and the conventional asset classes is stronger in the case of increased uncertainty. In other words, it can be stated that in the case of the Eurozone sovereign debt crisis and COVID-19 pandemic, the relationships between green and conventional assets are far closer than during more calm times. The analysis of network through the basis of the squared wavelet coherence shows to some extent the dynamic patterns of investor behavior and the relationship between assets over different periods of time: short-term (30 days), medium (180-360 days), and long-term (more than 720 days), which helps to obtain valuable data on the dynamics of investor behavior and correlations between assets over time.

Identification of the structure of dependence is imperative to designing the best investment portfolio towards greener products. With the evolution of interdependencies, especially in the scenario of economic shocks, integration of green financial products with conventional portfolios is likely to mitigate market risks and support sustainable investment strategies (Singh, 2026). Notwithstanding the advantages associated with the green bound of investment portfolios, various challenges must be considered. Investors have to deal with the fact that green finance is a dynamic field and they need to understand the relative performance of green bonds compared to traditional assets (Hussain & Shafique, 2026). The empirical evidence also indicated that green bond portfolios have low risk and high returns but the best portfolio strategy depends on the market context and risk preference of the investor.

A well optimized green portfolio should involve all aspects of the asset allocation process, regular rebalancing, including the incorporation of ESG and regular monitoring of market conditions. By these strategies, investors are able to restructure their portfolios in order to achieve financial as well as sustainability goals. The relationship between green bonds and other parts of the financial markets has received a lot of visibility in the recent academic works. Many approaches have been employed to study the relationships with the application of green bonds and conventional asset classes application advanced statistical processes like Quantile Vector Autoregression (QVAR) models. This is because such models are effective, particularly because they are not sensitive to outliers, hence enabling the one to carry out a good analysis of tail dependencies in various market conditions.

With a very high frequency (daily) prevalence of data, extensive literature review anticipates, the data prevalence has been utilized in almost 90 percent of scholarly papers on this topic. Whereas high-frequency data is suitable in a short-run analysis, green project characteristics that may take long durations of time indicate that low-frequency data (monthly or weekly) may be more suitable in making investment decisions concerning green bonds. This approach could make the investment more appealing to sustainability-focused investors (thefixedincome, 2024). Inclusion of the SDGs in the regulatory guidelines for green bonds is important to ensure that financial instruments are meeting global sustainability criteria. Integration not only makes it easier to monitor progress but also enhances accountability in selecting projects and funding (*The Evolution and Significance of the History of Green Bonds - Investology Hub*, 2024). It is important that investments made through green bonds are channeled towards climate actions and promote sustainable infrastructure and responsible consumption and production.

Environmental policy plays crucial role shaping the context of green bonds. Policymakers have a mandate to balance financial capital with environmental objectives through imposing regulations that favor issuance and regulation of green bonds. The Paris Agreement and other key agreements have helped to define bills that feature environmental benefits for investment. Policymakers can promote sustainable investment approaches and strategies through a supportive regulatory framework, aimed at sustainability and climatic sustainability objectives (Lateef, 2026). The interrelation between the green bonds and the other classes of assets forms a chain of areas to be investigated. The analysis of the processes of market interdependence (idiosyncratic, systematic, external) could be a source of data on what determines the presence of green bonds during periods of financial and economic crisis. Furthermore, the special features of green bonds, their hedging and safe haven attributes must be emphasized. Future research may employ some approaches developed to investigate asset classes possessing safe properties with the aim of enhancing the role of the same in investment portfolios.

Another one of the areas that can be studied is the diversification properties of green financial products. A portfolio and risk management approach may be beneficial in learning about green bonds, particularly when building portfolios that include both

green and traditional asset classes. Understanding the relationship between asset classes that is dependent on time, and understanding how investors of various time horizons might react to information, could lead to more efficient portfolio strategies. Financial markets are linked, so that will be a relationship with conventional asset classes and influence their valuation and risk control, together with “green bonds”. The dynamics of these market linkages during various economic scenarios and crises may yield valuable information for investors to understand and make informed decisions on resource allocation in the context of climate change. The findings of this study could help inform strategies to channel financial investments into green initiatives, and support the transition to a low carbon economy.

As the green bond market matures, it will be crucial to understand the characteristics and motives of the investors that have committed to buying green bonds. A research topic may be the differences between institutional investors (pension funds, insurance companies), and individual investors acting on ethical grounds (Green Bond Trends and Headwinds, n.d.). Understanding the different groups' perceptions of sustainability in investment decisions could help inform future product development and marketing strategies.

2. LITERATURE REVIEW

The aim of this study is to examine the complex interaction between green bonds and foreign financial markets, hoping to find ways to improve investors' investment portfolios for a sustainable future. This study (Mertzanis, 2023) uses a novel measure of energy policy effect diversity and IMF data (1991-2021) to calculate the effect it has on green bond issuance in 70 countries. Findings indicate higher energy policy diversity and other energy sources (nuclear and solar, in particular) spur the expansion of green bonds, in addition to environmental drivers such as lower emissions. However, the impacts of policy changes may have been lagged or nonlinear, with water infrastructure limitations creating challenges for nuclear energy. The results have implications for financial policies for green finance programmes and reveal cultural impacts on financial practices of countries. This study examines the relationship between green bonds (GB) and Environmental National Cap (ENC), green growth metrics (EC) and innovative financial mechanisms (INN_FM) for sustainability of environmental projects (EPs). It has discovered that GB has an important influence on EC, but not on ENC, while INN_FM has a small effect on GB issuance.

Multiplicity of dynamics is uncovered through the variance decomposition depending on regulatory and national context. The research provides key insights and policymaking suggestions for the sustainable optimisation role of green finance. A study by Mertzanis & Tebourbi (2025) has looked at the effect of geopolitical risk on green bonds issuance across 73 countries (from 2008 to 2021), taking into account three factors: deal, economic and institutional. It identifies a positive, nonlinear and time-lagged relationship which is confirmed using sensitivity and endogeneity analysis. Geopolitical factors positively correlated with green bond issuance on the whole. The study also highlights the significance of underwriters' networks and certain geopolitical jurisdictions in expanding the global markets.

This study (K.-H. Wang & Li, 2024) explores the relationship between green bonds (GBI) and (CP) in China (2014-2023) with the application of the rolling window causality method. Results reveal that GBI has a negative effect on CP, as it lowers the emission levels, and that in some periods, CP has a negative effect on GBI, due to policy-driven expectations. By improving returns, Stabilized CP boosts demand for green bonds. The results emphasize the importance of policy efforts to promote green finance and enhance the carbon market system for sustainable development. The other study (Mezghani et al., 2024) covers extreme connectivity in green bonds, stocks and commodities through the application of QVAR methodology. Results indicate that there are greater spillovers at extreme quantiles, with commodities being net transmitters and the Shanghai green bonds being net receivers. Gold bonds 'green' serve safe haven, provide diversification, especially from stocks and Brent markets. The information helps investors to fine-tune their portfolio strategy and hedging efforts through turbulent times.

This study (Mezghani et al., 2024) explores impact of oil prices (OP) on (GBI) from 2011 to 2021 using the (QQ) method. The short-term results reveal positive impacts of OP on GBI, implying that increased oil price spurs the growth of the green bond market. But in the medium and long term, OP has a negative impact on GBI because there is an oversupply of oil in the world and an increase in green energy profits. While green bonds are not an effective solution for the oil crisis, the GBI positively affects OP at all quantiles. The results inform the issuers and investors of green bonds aiming to make decisions based on the dynamics of oil prices. Another study (Jain et al., 2024) analyzes green bond market six Asian countries, identifying key enablers and inhibitors for its growth. A lack of formal institutional arrangements, underdeveloped bond markets, insufficient private participation, and the lack of international certification are obstacles to market development (Tumanggor et al., 2026).

The enhancement of capital markets, as experienced in developed economies, has the potential to stimulate domestic and/or foreign capital inflows and to support global climate pledges. The other study (Exploring the Synergies between Digital Finance and Clean Energy: A Case Study of Green Bond Spillover Effects | Environmental Science and Pollution Research, n.d.) concentrates on impact digital finance green bonds & clean energy based on China's data (2001-2019). Results of a time-varying causality test and dynamic spillover model show that there are spillover effects between clean energy and digital finance, and the impact of clean energy on the environment is -0.68% when using renewable energy. The research highlights rising volatility and causality significance concerning the shift from green energy sources to fintech financial platforms. In addition, it acknowledges that a stable regulatory framework and greater knowledge of the associated risks of clean energy investments crucial development of the green bond market.

A study (Jiang & Jia, 2022) characterizes the presence of extreme spillover impacts in green bond (GB) and clean energy (CE) markets (2011-2020) through MVMQ-CAViaR and the risk Granger causality frameworks. The asymmetric spillovers between GB and CE markets are strong and the upside spillovers are much greater than the downside spillovers. CE does not have any spillovers to GB. These findings are of great value in

the hands of investors, policymakers, and researchers. The urgency of climate-technology revolution to achieve the goals of the Paris Agreement is another study (Aghion et al., 2022) which records lower low-carbon innovation in the EU than its peers. It also establishes that in a model of directed technical change, most of the cross-country gap in green patents per capita can be attributed to carbon tax, R&D and venture capital investment changes. The article concludes that the governments are the key actors of green innovation and central banks are marginal.

The other study (D. Wang et al., 2024) examines the impact of cleaner bonds in financing clean energy projects and creating sustainable economic revival through fuzzy analytical hierarchy procedures. The results prove that cleaner bonds increase clean energy expenditures by 4.9 percent per year and prompt the development of the green economy by 17 percent per year. Energy performance contracts are actual alternatives to conventional financing instruments. The research highlights strategic investment of cleaner bonds by the government and industries to propel sustainability, energy conservation and demands stakeholders to restore balance of the law. In this research (Patel et al., 2022) the authors explore the cleaner bonds issue which can be used to fund clean energy projects and promote sustainable economic regeneration by the means of fuzzy analytical hierarchy processes. Empirical evidence indicates that cleaner bonds increase clean energy investments by 4.9% per annum and contribute to a growth rate of 17% per annum of a green economy. Financial performance contracting has become one of the possible alternatives to the conventional financing. The research suggests that strategic public-private investments in cleaner bonds are necessary to improve the sustainability and energy efficiency and encourage the stakeholders to think about the legal implications.

This study (Zhao & Duan, 2023) implements a DEA and system GMM study that examines the impact of financial performance on the energy transition of energy efficiency (2017-2022) in Asia. The outcomes indicate the impact of the green financing in causing a 30 percent transition between traditional and renewable energy sources where green power and hydropower projects exhibit a high rate. Green bond issuance is induced by sustainable industrial and agricultural growth, with a need for improved finance systems, new efficiency, and long-term technological infrastructure. Being the first study to respond to green finance in Asia's renewable energy's transition, it provides practical suggestions for managing sustainable energy transitions.

The intricacies of Green Bonds and the complex relationships with economic and environmental variables are discussed in the literature. The green bond issuance is dependent on many factors such as energy policy, geopolitical risks, carbon-pricing, and linkage with other financial markets, research shows. The research described above is about the dynamics of the green bond market, but more research is needed to tackle future issues such as the development of green bond standards, transparency and accountability, and the assessment of the impact of green finance on the realization of climate goals. A deep insight into these complexities is critical to policymakers, investors, and issuers to harness green bonds as an effective instrument for sustainable development and low-carbon economic transformation.

3. METHOD

3.1 Data collection

This study utilized 2014-2024 daily time series data. The following financial indicators were procured to be used as independent variables:

Table 1: Financial Index Name with Description

Financial Index Name	Description
S&P Green Bond Index	Represents performance of global green bond market.
S&P GSCI Commodity Index	Tracks performance of broad basket of commodities.
MSCI World Index	Represent performance of large or mid-cap equities across developed markets.
S&P 500 Composite Index	Tracks the performance of 500 large-cap U.S. companies.
DAX Index	Represents the performance of 30 largest companies listed on Frankfurt Stock Exchange.
China Shanghai Composite Stock Market Index	Tracks the performance of all A-shares and B-shares listed on the Shanghai Stock Exchange.
S&P Clean Energy Index	Tracks the performance of companies involved in renewable energy with clean technology.
Bitcoin Daily Price Index	Measures daily closing price of Bitcoin.

Source: Author designed

The data for each index is from reputable financial information sources, and respective links are provided in references (*Bitcoin Historical Data - Investing.Com*, n.d.; *DAX Historical Data (GDAXI) - Investing.Com*, n.d.; *MSCI World Index*, n.d.; *Shanghai Composite Historical Data (SSEC) - Investing.Com*, n.d.; *S&P 500® | S&P Dow Jones Indices*, n.d.; *S&P Global Clean Energy Index | S&P Dow Jones Indices*, n.d.; *S&P Green Bond Index*, n.d.; *S&P GSCI | S&P Dow Jones Indices*, n.d.).

3.2 Data preprocessing

The cleaning of the data entailed several steps with the aim of ensuring accuracy and consistency of dataset used in the analysis. First, data selection was obtained by querying and retrieving each of the indices' (S&P Green Bond Index, S&P GSCI Commodity Index, etc.) daily time series data from their respective databases. Data were retrieved for the 2014-2024 range. Second, the consistency of the data was achieved. The alignment of dates was essential and all the data series were therefore verified and brought to the same dates in order to be free to make valid temporal comparisons. Missing values: The missing values were addressed by removing the rows containing a missing value in one of the columns. This was to make sure that there was

integrity of the data and also to prevent possible introduction of biases with the help of imputation techniques.

Lastly, the data format was standardized so that there would be uniformity in all the columns. It did this in normalizing date format (e.g., YYYY-MM-DD) and numeric format on all data series. Through the completion of these processes, the data were already prepared to be analyzed further and therefore, to guarantee its accuracy, reliability and appropriateness to the intended purpose of research. The completion of the analytical process was informed by the goal of the data cleaning process that sought to provide the accuracy, consistency, and reliability of financial time series data used. This was done in a cyclical process which entailed close selection, alignment, transformation, and standardization of the data.

4. FINDINGS

4.1 Analysis 1

4.1.1 Data Preparation

Standardization: Before calculating correlations, it's crucial to standardize the data. This involves converting each data point to a z-score, which removes the influence of different units and scales. The z-score for a data point is calculated as:

$$z = (x - \mu) / \sigma \quad (1)$$

where: x is individual data point, μ is mean of dataset, σ is the standard deviation of dataset.

4.1.2 Correlation Coefficient Calculation

Pearson Correlation Coefficient: Most common correlation coefficient is Pearson correlation coefficient (ρ), in which measures linear relationship between two variables. It ranges from -1 to +1, where: -1 indicates a perfect negative correlation (as one variable increases, the other decreases), 0 indicates no correlation, +1 indicates a perfect positive correlation (as one variable increases, the other also increases).

Calculation: The Pearson correlation coefficient between two variables, X and Y, can be calculated as:

$$\rho(X, Y) = \text{Cov}(X, Y) / (\sigma_X * \sigma_Y) \quad (2)$$

where: $\text{Cov}(X, Y)$ is the covariance between X and Y, σ_X is the standard deviation of X and σ_Y is the standard deviation of Y

The covariance between two variables is calculated as:

$$\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] \quad (3)$$

where: $E[]$ denotes the expected value (mean)

4.1.3 Correlation Matrix Construction

A report correlation coefficient can be tabulated in a structured correlation matrix to show all the possible pairs of variables in a data set. The two variables are tested in each table entry to determine the strength of the correlation between them to provide information about the linear correlations of the two variables. The elements of the diagonal of the matrix are necessarily 1.0, which means that all variables are perfectly correlated with themselves. The matrix is also usually symmetric, that is, correlation coefficient between variable X and variable Y is equal between Y and X. This levels of

correlation and symmetry makes the correlation matrix have a valuable application in interdependence analysis of variables in a data set.

4.1.4 Interpretation and discussion

Of course, some rudimentary steps can be used to carry out an adequate analysis of a correlation matrix. To begin with, strong correlations are obtained by seeking correlation coefficients that are either equal to 1 or -1 as this would indicate a strong positive or negative linear relationship among variables. Secondly, we can combine variables with high correlation to facilitate observation of their internal dynamics and their effects on market dynamics. Thirdly, one can create a heatmap or a correlation plot in order to visualize the correlation matrix. These graphical tools may be utilized in helping to observe patterns, trends as well as strong correlations in the data thereby improving the process of analysis.

In a bid to explore the associations among various financial indices, the process begins by normalizing the value of the returns of each of nine indices such as Green Bond Index of S&P and Commodity Index of GSCI on a daily basis. This is then succeeded with a Pearson correlation matrix which generates a 9x9 correlation matrix whose correlation coefficients are the straight-line associations between all conceivable combinations of indexes. This is then filtered to detect eye-catching positive or negative correlations to explore the linkages between Green Bonds and other classes of assets and clustering of assets according to their correlation behaviors. For instance, some commodities can be highly correlated while others can be weakly or even negatively correlated. For better visualization, a heatmap of the correlation matrix can be built, which graphically displays the correlation relationships and patterns between the various asset classes.

4.2 Analysis 2: Rolling window base correlation method

Rolling Window Wavelet Correlation Model: A Comprehensive Overview

The wavelet correlation method of rolling window was employed to explore dynamic correlations between the S&P Green Bond Index and other financial indices, e.g., the S&P GSCI Commodity Index and the MSCI World Index, etc. The approach attempted to explore temporal changes in correlations that are difficult to quantify by the conventional Pearson's correlation method.

4.2.1 Wavelet Transform

The wavelet transforms mathematical technique used the decomposition of a time series into different frequency components. Unlike Fourier transform, which used measure the entire time series as a whole, the wavelet transform yields a time-frequency representation and allows for the detection of localized patterns and time-varying changes in frequency. The continuous wavelet transform (CWT) was applied to all the time series data that were studied. This CWT yields a time-frequency representation of the data, which represents the distribution of signal energy over a set of frequencies at different temporal locations. Two of the most widely used wavelet functions are the Morlet wavelet and the Mexican hat wavelet.

4.2.2 Rolling Window Analysis

A rolling window method was applied to the time series data. This was achieved by dividing the data into overlapping fixed window length windows. The wavelet

transform for both the time series was calculated for each window. The wavelet coefficients of the two series were then multiplied to obtain cross-wavelet transform. Then squared magnitude of cross-wavelet transform was calculated averaged across all the frequencies to obtain the wavelet coherence. The wavelet coherence was calculated for each window, and this provided a time series of wavelet coherence values. This time series obtained reflects the dynamic evolution of the two-time series correlation.

4.2.3 Correlation Analysis and Interpretation

Wavelet correlation analysis with the rolling window emphasized the dynamic nature existing correlations Green Bonds & other financial indexes. Large variation in wavelet coherence throughout the analysis period identified high and low correlation phases, which meant the relationships were not static but fluctuated with time. Wavelet transforms provided crucial information regarding the frequency components driving the correlations. This facilitated the identification of short-term & long-term relationships among assets. Analysis was aimed at identifying high positive or negative correlation periods, which can be of value in portfolio diversification, risk management, and trading.

4.2.4 Mathematical Representation (Simplified)

- Let $x(t)$ and $y(t)$ represent two time series.
- Wavelet transform of $x(t)$: $W_x(a, b)$
- Wavelet transform of $y(t)$: $W_y(a, b)$
- Cross-wavelet transform: $W_{xy}(a, b) = W_x(a, b) * W_y^*(a, b)$
- Wavelet coherence: $|S_{xy}(a, b)|^2 / (S_{xx}(a, b) * S_{yy}(a, b))$, where $S_{xy}(a, b)$ is the cross-spectrum and $S_{xx}(a, b)$, $S_{yy}(a, b)$ are the auto-spectra.

Key Considerations:

Choice of Wavelet Function: Choice the wavelet function significantly affect results of the analysis.

Window Size: Choice the window size is of particular significance. A larger window size can effectively counteract short-term changes; however, it can fail to capture rapid changes in correlation. In this study, we use a mixed window size to obtain a proper balance between smoothing and rapid change detection capability.

Interpretation: Close attention must be paid to Interpretation of the results, given the limitations inherent in the wavelet transform as well as in understanding financial markets.

4.3 Experimental Setup and Results

4.3.1 Correlation of Green Bonds with International Markets:

Overall Observations:

The correlation matrix defines broad associations between various classes of assets. We can notice that there exist both positive and negative correlations that imply diverse responses to market change.

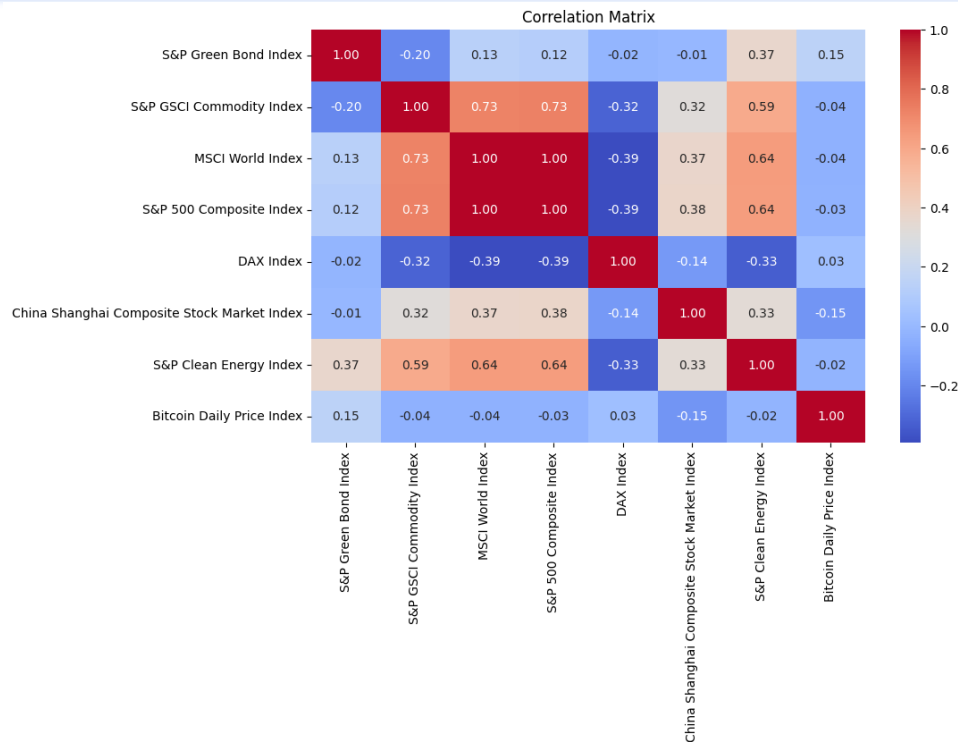


Figure 1: Correlation of Green Bonds with International Markets

Source: Author designed

The correlation with other financial indexes is different for every Green Bond, indicating the extent to which they have been linked to the international financial system, and to which they could be useful to diversification. Partially, there are moderate correlations between Green Bonds and the major indices, such as the S&P GSCI Commodity Index (0.13), MSCI World Index (0.12), and S&P 500 Composite Index (0.12). These correlations show that Green Bonds are correlated with commodities, global equity markets and U.S. markets, highlighting the increasing relevance of Green Bonds in diversified investment portfolios and their linkage to broader markets. Further, a materially higher positive correlation with the S&P Clean Energy Index (0.37) is indicative of the underlying relationship between Green Bonds and the sustainability-driven clean energy industry, indicative of their shared environmental objectives.

On other hand, Green Bonds have weak or no correlations with the majority of indices, including the DAX Index (-0.02) and the China Shanghai Composite Stock Market Index (-0.01), indicating a weak relationship with the German and Chinese equity markets. In addition, the weak positive correlation with Bitcoin (0.15) indicates a limited relationship with this very volatile asset class. These lower correlations indicate Green Bonds could provide good diversification benefits to investors, especially portfolios that aim to minimize exposure to some market risks.

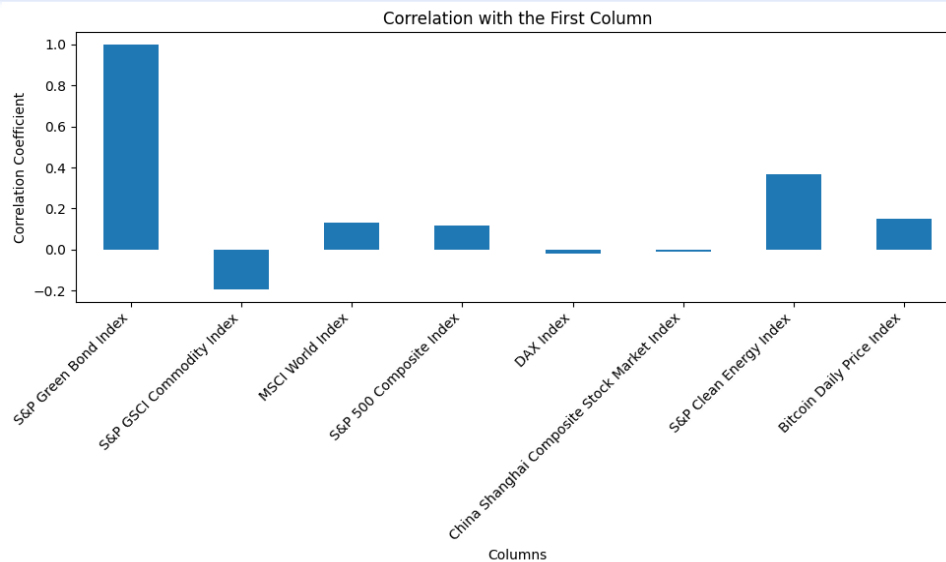


Figure 2: Correlation coefficients between the S&P Green Bond Index and several other financial market indices

Source: Author designed

The graphical representation illustrates the correlation coefficients between the S&P Green Bond Index and various other financial market indices. A correlation coefficient quantifies the strength and direction of the linear relationship between two variables. A coefficient of 1 indicates a perfect positive correlation, -1 indicates a perfect negative correlation, and a coefficient of 0 signifies no correlation at all. The correlation visualization portrays a collection of interrelations among the S&P Green Bond Index and other indices of the financial market. Worth noting is a high positive correlation between the green bond index and the S&P GSCI Commodity Index, indicating that commodity price movements significantly influence the performance of green bonds. On the other hand, the correlation rates with other large stock market indices like the MSCI World Index and the S&P 500 Composite Index are usually low implying zero interdependence level with the stock market in general. Moreover, the visualization demonstrates the moderate positive relationship with the S&P Clean Energy Index, which implies the correlation between green bonds performance and clean energy industry. The implication of these results is that green bond can give diversification advantages to investment portfolios depending on their special correlation patterns with other asset classes.

4.3.2 Rolling Window Wavelet Correlation Analysis

Wavelet heatmap is a graphical depiction of the relationship between two time series of various scales or frequencies of the wavelet. It measures the relationship between the time series of the various periods by decomposing the data into its root frequencies using the wavelet decomposition. The associations are then represented using color gradation in order to reflect the amount and the direction of association. The colors that are usually used (red color) are used to indicate positive association whereas (blue or green color) is used to indicate negative association. The intensity of colors is also an

indicator of increasing association. With the pattern in the heatmap, researchers can determine which time series or frequency bands have significant co-movement or anti-movement between the series.

The findings are beneficial in explaining dynamic relationships between the time series in addition to identifying the possible cause-and-effect relationship. The visualization represents the pair-wise rolling window wavelet correlation coefficients of financial market indices. The color scale, ranging from red to green, indicates the strength and direction of association at different temporal frequencies. Red indicates a strong positive association, while green indicates a strong negative association. The analysis considers four wavelet scales (D1-D4), which correspond to time intervals of around 30 days, 180 days, 360 days, and 720 days, respectively. This arrangement allows for an investigation of the correlation dynamics between the indices over a variety of time horizons.

4.3.3 S&P Green Bond Index with S&P GSCI Commodity Index

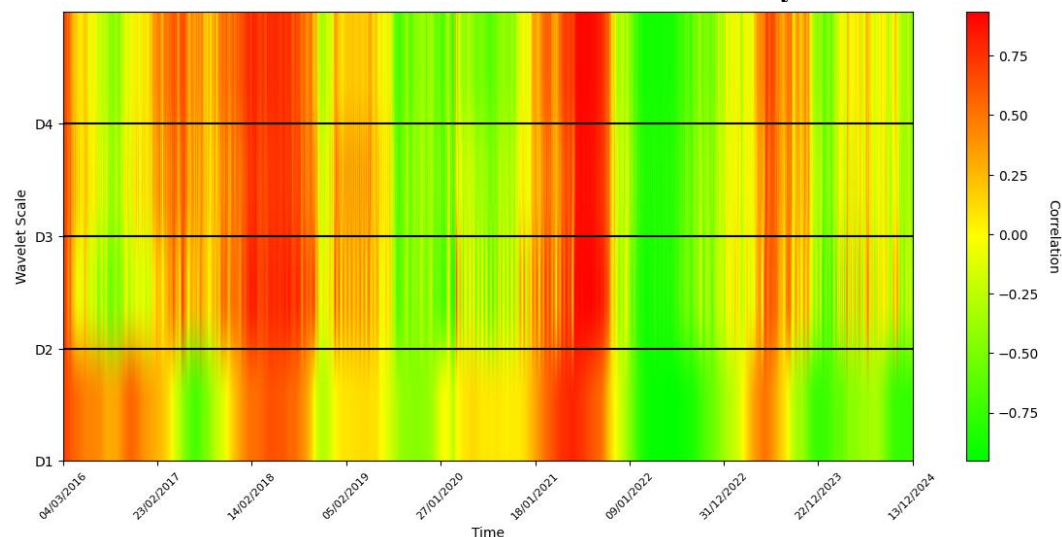


Figure 3: S&P Green Bond Index with S&P GSCI Commodity Index

Source: Author designed

Wavelet coherence plot for S&P Green Bond Index and S&P GSCI Commodity Index for time periods and various wavelet scales (D1 - D4). Correlations are strong (shown in red areas) around the year 2020, where there is more consistency. Weak or negative correlations (green areas) are primarily scattered in the dataset, especially before the year 2018 and after the year 2022, where there is less consistency or negative relationships. The trend is time and scale-varying, showing the dynamic relationship between the two indices.

4.3.4 S&P Green Bond Index with MSCI World Index

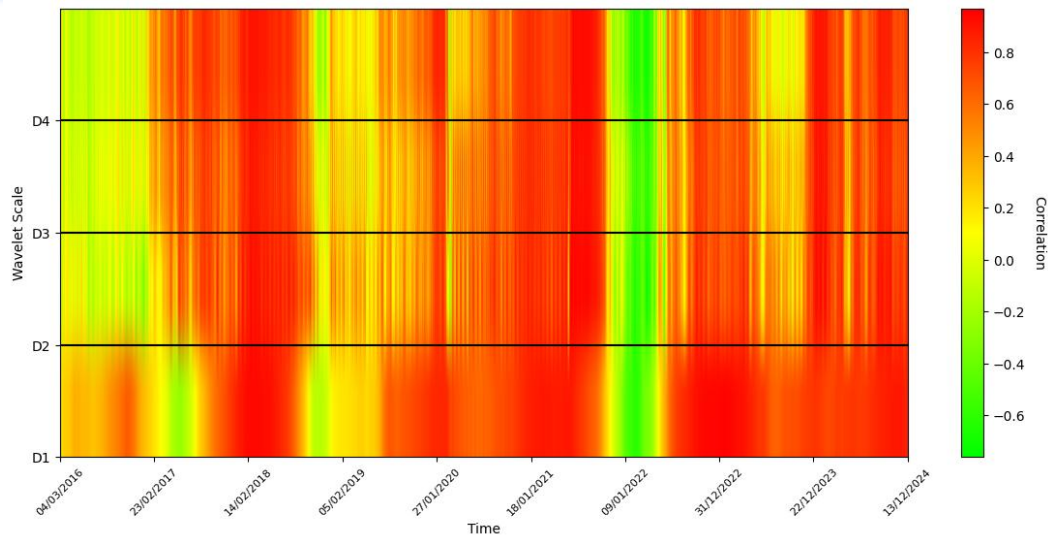


Figure 4: S&P Green Bond Index with MSCI World Index

Source: Author designed

The wavelet coherence between the S&P Green Bond Index and the MSCI World Index shows the patterns of the wavelet scale from D1 to D4 over time. In almost all places, there are parts with high positive correlations (colored red) that cover a significant part of the period since the beginning of the dataset (e.g., from 2011 to 2021) and that are present in a substantial part of the period since 2000 (e.g., from 2019 to 2022), which means that there is a strong and persistent correlation between these indices in these parts of the period. Regions with lower values, or negative values, shown in a green color, are found randomly distributed and in many cases from earlier years (2016) and again in some years (2023) indicating less or inverse relationships in these years. The volatility shows the temporal and spatial dynamics of the correlation of these indices.

4.3.5 S&P Green Bond Index with S&P 500 Composite Index

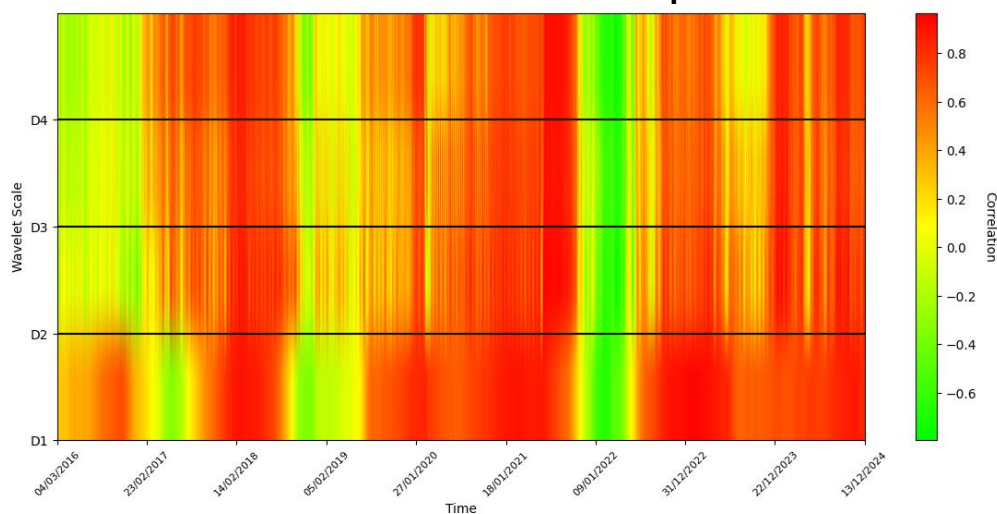


Figure 5: S&P Green Bond Index with S&P 500 Composite Index

Source: Author designed

It displays wavelet coherence plots for the relationship between S&P Green Bond Index and S&P 500 Composite Index, in which fluctuations along the wavelet scale (D1 – D4) are indicated as well as across time. The colour bar shows the strength of the correlation, where red signifies a strong positive correlation, green signifies a strong negative correlation and yellow/orange signifies a moderate correlation. The overall picture is characterized by a dominant strong positive correlation (the red areas) which is obvious over most of the observation period, especially in 2018-2022, indicating a general synchronization between the two indicators during these years. There are downward or inverse correlations (green areas) that appear intermittently before the year 2017 and occasionally after the year 2022. This is an indicator of the time and space dependency of the association of these indices.

4.3.6 S&P Green Bond Index with DAX Index

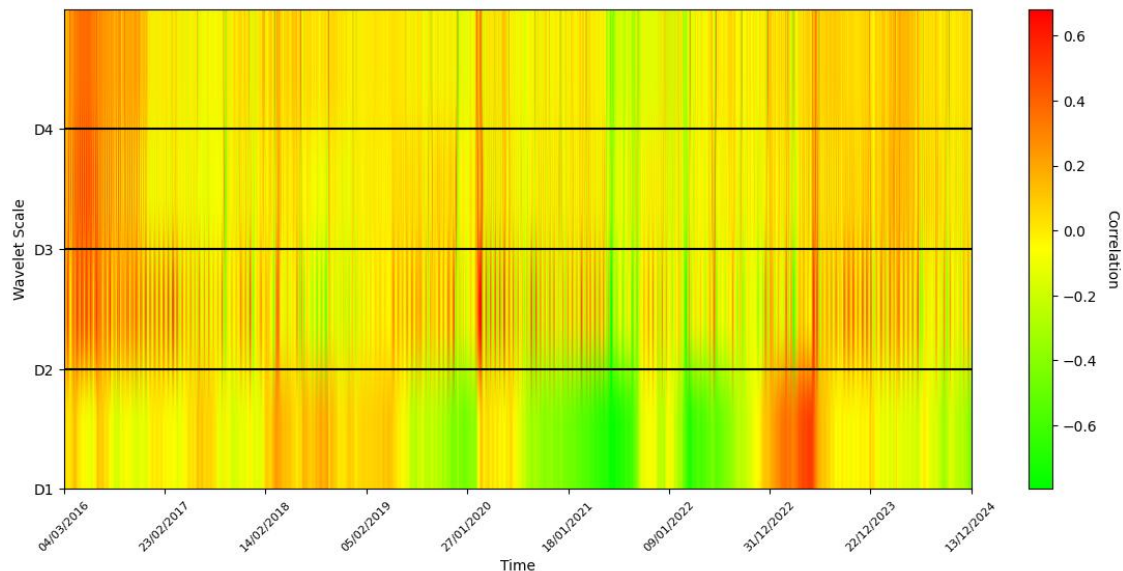


Figure 6: S&P Green Bond Index with DAX Index

Source: Author designed

The wavelet correlation heatmap displays the correlation between Green Bond Index and DAX Index at the different wavelet scales (D1–D4) and time, with red colors indicating strong positive correlation, green colors strong negative correlation, and yellow colors weak or no correlation. Interestingly, there are periods of strong positive correlation around September 2019 and mid-January 2022 and strong negative correlations in mid-2018 and early-2021. The value of the correlation varies over time and it is alternate with high positive and high negative values indicating dynamic dependency between the analyzed time-series data. This visualization can help to find temporal patterns and variations at various frequency scales.

4.3.7 S&P Green Bond Index with China Shanghai Composite Stock Market Index

The wavelet coherence is plotted over time and wavelet scale (D1 to D4) for the S&P Green Bond Index and the China Shanghai Composite Stock Market Index. One conspicuous characteristic is that there are distinct periods of high positive correlation

(red regions), especially in the vicinity of 2018 and 2021. The time periods indicate high correlation between the movements of both the indices in specific time. On the other hand, there are negative or weak correlations (green areas) at some times and scales in 2019 (weak or inverse relationships between the indices at certain times and scales). The trend illustrates the relationship between the S&P Green Bond Index and the China Shanghai Composite Stock Market Index and the dynamic and scale-dependent nature of the relationship during the analyzed time horizon.

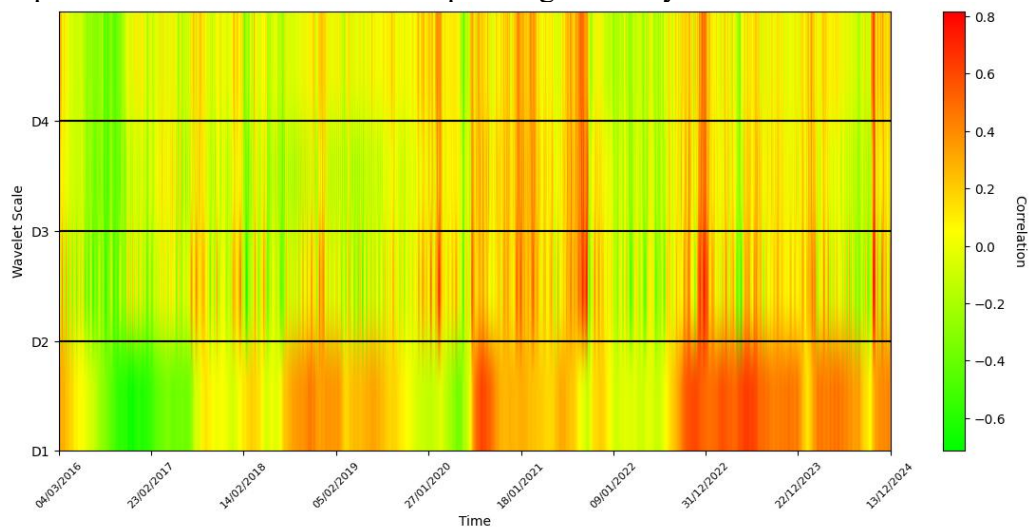


Figure 7: S&P Green Bond Index with China Shanghai Composite Stock Market Index

Source: Author designed

4.3.8 S&P Green Bond Index with S&P Clean Energy Index

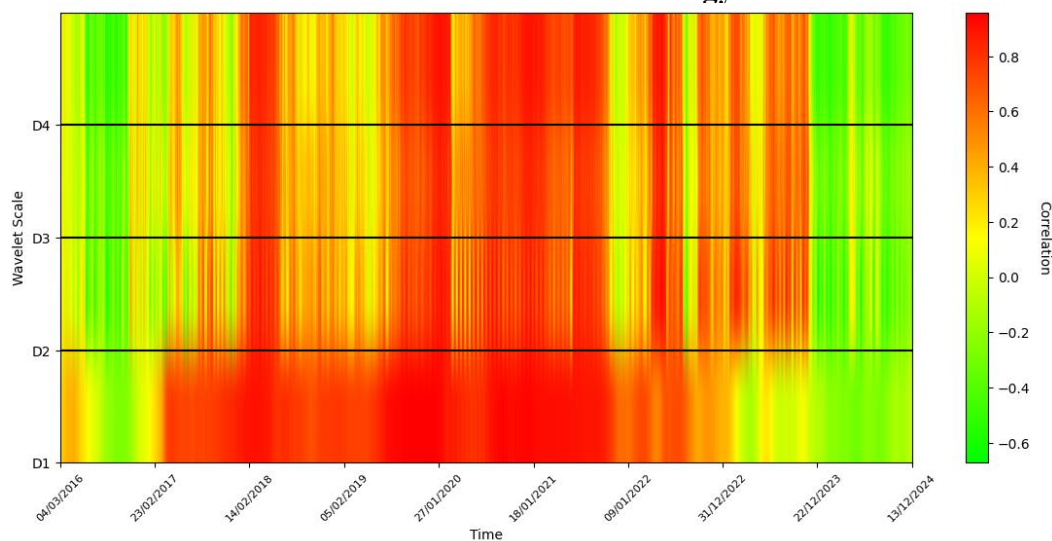


Figure 8: S&P Green Bond Index with S&P Clean Energy Index

Source: Author designed

The wavelet coherence analysis of S&P Green Bond Index and S&P Clean Energy Index indicates a significant and enduring correlation, particularly after mid-2017, and

there are fleeting moments of negative correlations in the form of green patches around mid-2015 and early 2016. As can be seen, there is a stronger positive correlation within the lower scales of the wavelet (D1 and D2) suggesting a strong relationship at the higher frequencies, and a relatively stable positive correlation within the higher scales (D3 and D4). This visualization indicates that more and more time has passed since the analyzed signals started to have more similar fluctuations (synchronized) and less time since they started to have opposite fluctuations (anti-synchronized). High correlation is not unexpected as both indices are related to clean energy and have significant similarities. Due to this similarity, we included both in our analysis.

4.3.9 S&P Green Bond Index with Bitcoin Daily Price Index

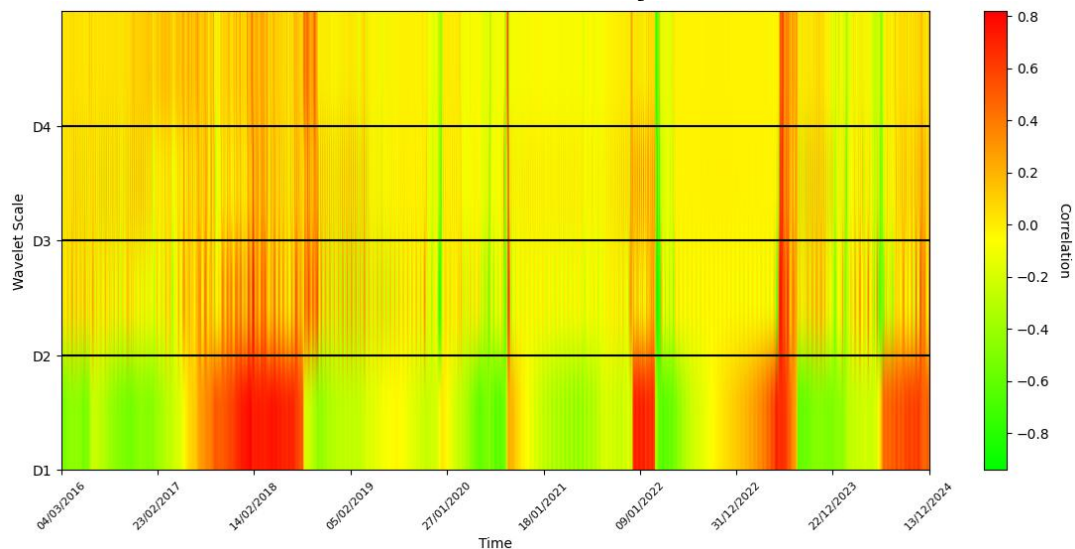


Figure 9: S&P Green Bond Index with Bitcoin Daily Price Index

Source: Author designed

The wavelet correlation S&P Green Bond Index with Bitcoin Daily Price Index heatmap also continues to show the same trend: strong positive correlation (red) and strong negative correlation (green) between the different wavelet scales (D1-D4) and time. In this connection, the variations of D1 and D2 scales are more prominent and there are peaks of positive correlation in the years 2017, 2021 and 2022, and peaks of negative correlation in the years 2018 and 2020. Lower frequency components are more temporally stable (less variation) and the higher wavelet scales (D2 and D3) are more stable distributions. These results indicate a dynamic pattern correlation change and the use of wavelet analysis in tracking changing patterns.

The research method used is a wavelet transform, which was used to analyze the dynamic correlation of the S&P green bond index and other financial indexes, including the S&P GSCI commodity index and the MSCI world index. The method enabled time series to be broken down into separate frequency components, making it easier to identify time-varying correlations. The results highlighted that there are different levels of correlation between Green Bonds and other asset classes. Correlation between the

S&P GSCI Commodity Index and the S&P Clean Energy Index was strong while the correlation between the S&P GSCI Commodity Index and leading stock market indices was low. The wavelet analysis also revealed the dynamic aspects of the associations, including high and low correlation at different time scales.

5. CONCLUSION

The green bond dependence relationship that is unique to green bonds and the mature finance market provides interesting insights into the functioning of green bonds and how they can be leveraged for sustainable investment and portfolio diversification. Based on this research, we find that the dynamics between green bonds and traditional asset classes such as stocks, commodities, cryptocurrencies and conventional bond markets strengthen during economic stress as they address the growing importance of complementing the portfolio's resilience in times of market turbulence. Such methods as wavelet coherence and rolling window analysis reveal the temporal and frequency evolution of such interlinkages. Our results show that green bonds have varying degrees of correlation with other financial assets. While their link with traditional bonds is mostly constant owing to the common fixed-income nature, their link with commodities and equities is variable with macroeconomic variables, policy changes, and change in sentiment. Furthermore, the relationship of green bonds and cryptocurrencies seems less tight and less stable, highlighting the speculative demand of digital currencies, which contrasts with the demand for green fixed income securities that has been driven by stability.

Green bonds offer diversification and safe haven benefits, but market trends, regulations and investor sentiment must be used wisely when building an investment portfolio. Green bond market continues to grow and issues such as liquidity gap, greenwashing and the standardization of green bonds are playing a key role in ensuring market longevity. Future research is needed to expand the standards of green bonds, increase transparency in the standards and its long-term impact on achieving global sustainability goals. These insights will be crucial for policy-making, institutional investment and market participants to enhance asset allocation and promote sound low-carbon finance systems (Machava & Ribeiro, 2026). Through more efficient regulatory systems and the development of sustainable finance programs, green bonds can play major role in taking the world towards a greener, more environmentally aware global economy.

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